

Vector Derivatives

Cartesian

$$\begin{aligned}d\mathbf{l} &= dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}} \\ \nabla f &= \hat{\mathbf{x}} \frac{\partial f}{\partial x} + \hat{\mathbf{y}} \frac{\partial f}{\partial y} + \hat{\mathbf{z}} \frac{\partial f}{\partial z} \\ \nabla \cdot \mathbf{F} &= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \\ \nabla \times \mathbf{F} &= \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \hat{\mathbf{x}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) \hat{\mathbf{y}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \hat{\mathbf{z}}\end{aligned}$$

Cylindrical

$$\begin{aligned}d\mathbf{l} &= ds \hat{\mathbf{s}} + s d\phi \hat{\boldsymbol{\phi}} + dz \hat{\mathbf{z}} \\ \nabla f &= \hat{\mathbf{s}} \frac{\partial f}{\partial s} + \hat{\boldsymbol{\phi}} \frac{1}{s} \frac{\partial f}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial f}{\partial z} \\ \nabla \cdot \mathbf{F} &= \frac{1}{s} \frac{\partial(sF_s)}{\partial s} + \frac{1}{s} \frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z} \\ \nabla \times \mathbf{F} &= \left(\frac{1}{s} \frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z} \right) \hat{\mathbf{s}} + \left(\frac{\partial F_s}{\partial z} - \frac{\partial F_z}{\partial s} \right) \hat{\boldsymbol{\phi}} + \frac{1}{s} \left(\frac{\partial(sF_\phi)}{\partial s} - \frac{\partial F_s}{\partial \phi} \right) \hat{\mathbf{z}}\end{aligned}$$

Spherical

$$\begin{aligned}d\mathbf{l} &= dr \hat{\mathbf{r}} + r d\theta \hat{\boldsymbol{\theta}} + r \sin \theta d\phi \hat{\boldsymbol{\phi}} \\ \nabla f &= \hat{\mathbf{r}} \frac{\partial f}{\partial r} + \hat{\boldsymbol{\theta}} \frac{1}{r} \frac{\partial f}{\partial \theta} + \hat{\boldsymbol{\phi}} \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \\ \nabla \cdot \mathbf{F} &= \frac{1}{r^2} \frac{\partial(r^2 F_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\sin \theta F_\theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial F_\phi}{\partial \phi} \\ \nabla \times \mathbf{F} &= \frac{1}{r \sin \theta} \left(\frac{\partial(\sin \theta F_\phi)}{\partial \theta} - \frac{\partial F_\theta}{\partial \phi} \right) \hat{\mathbf{r}} + \left(\frac{1}{r \sin \theta} \frac{\partial F_r}{\partial \phi} - \frac{1}{r} \frac{\partial(r F_\phi)}{\partial r} \right) \hat{\boldsymbol{\theta}} \\ &\quad + \frac{1}{r} \left(\frac{\partial(r F_\theta)}{\partial r} - \frac{\partial F_r}{\partial \theta} \right) \hat{\boldsymbol{\phi}}\end{aligned}$$

Vector Identities

Triple Products

$$(1) \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$(2) \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

Product Rules

$$(3) \nabla(fg) = f(\nabla g) + g(\nabla f)$$

$$(4) \nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$$

$$(5) \nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$$

$$(6) \nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$(7) \nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$$

$$(8) \nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$$

Second Derivatives

$$(9) \nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$(10) \nabla \times (\nabla f) = 0$$

$$(11) \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$