Electricity and Magnetism I (PHY 321)

Biot-Savart Law problems

Problem 1 Consider a straight segment of wire with length L. The segment is located along the z-axis, with one end at $\mathbf{r} = \frac{L}{2}\hat{\mathbf{k}}$ and the other end at $\mathbf{r} = -\frac{L}{2}\hat{\mathbf{k}}$. There is a current I flowing in this wire segment in the positive z-direction. (Strictly speaking, it is not possible to have a current flow in a wire segment, because the current has no place to go when it gets to the end. However, this is a useful result because we can make closed loops like squares, hexagons, and things like that out of wire segments.) Use the Biot-Savart law to find the magnetic field at an arbitrary point $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$.

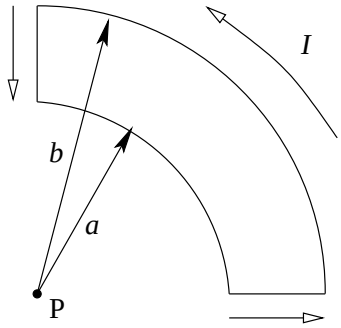
Problem 2 Consider a circular current loop with radius R carrying current I. Find the magnetic field produced at the center of the loop.

Problem 3 Find the magnetic field on the z-axis produced by a circular current loop with radius R lying in the xy-plane, centered at the origin, carrying current I.

Problem 4 Using the result for the magnetic field produced by a line segment of current, find the magnetic field on the z-axis produced by a square current loop of side L, lying in the xy-plane, centered at the origin. The current loop carries a current I.

Problem 5 Using the result for the magnetic field produced by a line segment of current, find the magnetic field on the z-axis produced by a triangular current loop of side L, lying in the xy-plane, centered at the origin. The current loop carries a current I.

Problem 6 Find the magnetic field at point P produced by the current loop shown in the figure. The current loop carries current I, and is composed of two straight segments and two quarter-circle arcs of radii a and b. The loop lies in the xy-plane, and the positive z direction is out of the paper.



Problem 7 About how much current do you need in a wire to get a magnetic field 1 m away that is as big as Earth's magnetic field?

Problem 8 What size magnetic field can be created inside a solenoid? Pick some reasonable values and do the calculation.

Problem 9 What size magnetic field can be created with a Helmholtz coil? Pick some reasonable values and do the calculation.

Problem 10 Consider two infinitely long straight wires parallel to the z-axis. One wire intersects the x-axis at $x = a/2$, and carries a current I in the positive z-direction. The other wire intersects the x-axis at $x = -a/2$, and carries a current I in the negative z-direction. Give an expression for the magnetic field produced by these two wires at a point $x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$. Express your final answer for the magnetic field in Cartesian coordinates.

Problem 11 Find the magnetic field produced by a square loop of wire with side length L carrying a current I.

Problem 12 Take the limit of the previous result as $L \to 0$ holding the magnetic moment $m = IL^2$ constant. This gives the magnetic field produced by an ideal magnetic dipole.

Problem 13 What is the fundamental source of magnetic field? Is it current, or is it magnetic dipole moment? Any magnetic field can be thought of as having been created by a current density via Ampere's law. Can any magnetic field be thought of as having been created by magnetic dipole moment (magnetization)?

Problem 14 Consider a circular loop of wire with radius R centered at the origin in the xy-plane, carrying current I in the counter-clockwise direction (viewed from a point on the positive z-axis). Find the magnetic field produced by this loop at position $x\hat{\mathbf{i}} + z\hat{\mathbf{k}}$. Give separate expressions for B_x, B_y, and B_z. Your expressions should include integrals with appropriate limits, without any unit vectors. You do not need to evaluate any of these integrals, but if you can argue that an expression should be zero on physical or mathematical grounds, please do so.

Problem 15 Consider a square loop of wire with side length L centered at the origin in the xy-plane, carrying current I in the counter-clockwise direction (viewed from a point on the positive z-axis). Find the magnetic field at $\mathbf{r} = L\hat{\mathbf{i}}$.