Electricity and Magnetism I (PHY 321)

Capacitor problems

Problem 1 A capacitor with capacitance C is charged to an electric potential V_0. The charged capacitor is then connected in parallel with a resistor (resistance R). Let I be the current that flows around this circuit. Let $Q(t)$ denote the charge on the capacitor as a function of time. From Ohm's law, we have

$$V = IR$$

where V is the voltage across the resistor. For the capacitor, we have

$$Q = CV$$

where Q is the charge on the capacitor, and V is the voltage across the capacitor. The voltage across the capacitor is the same as the voltage across the resistor, so

$$\frac{Q}{C} = IR.$$

Using

$$I = \frac{dQ}{dt},$$

we get a differential equation

$$\frac{Q}{C} = \frac{dQ}{dt}R.$$

We can rewrite this as

$$\frac{dQ}{dt} - \frac{1}{RC}Q = 0.$$

This is a first-order, linear, homogeneous, ordinary differential equation. It's solution is

$$Q(t) = Q_0 e^{t/RC}.$$

Substituting the initial condition $Q_0 = Q(0) = CV_0$, we have

$$Q(t) = CV_0 e^{t/RC}.$$

The current flowing through the resistor is

$$I(t) = \frac{V_0}{R} e^{t/RC}.$$

The power dissipated by the resistor is

$$P = I^2 R = \frac{V_0^2}{R} e^{2t/RC}.$$

Suppose we have a voltage of 1.5 V (from a AA battery, say), a capacitance of 0.01 μF, and a resistance of 1 kΩ. (These are all parts you could find in the Electronics Lab.) After 0.2 ms, the resistor is dissipating 500,000 GW, about the output power of half a million nuclear power plants. What went wrong with our analysis?

Problem 2 Find the capacitance per unit length for a cylindrical capacitor consisting of a long conducting cylindrical shell of radius a and a coaxial long conducting cylindrical shell of radius b. Assume $a < b$. (Hint: A good way to get started is to assume that one of these shells has a charge per unit length λ on it, and that the other shell has a charge per unit length $-\lambda$.)

Problem 3 Find the capacitance for a spherical capacitor consisting of a conducting spherical shell of radius a and a concentric conducting spherical shell of radius b. Assume $a < b$. (Hint: A good way to get started is to assume that one of these shells has a charge Q on it, and that the other shell has a charge $-Q$.)