A Catalog of Fields, Paths, Surfaces, and Volumes

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1.1 Scalar fields expressed in Cartesian coordinates

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1.1.2 Electric potential produced by a point charge q at the origin

$$V(x, y, z) = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{x^2 + y^2 + z^2}}$$

1.1.3 $f(x, y, z) = x^4 + y^4 + z^4$

1.1.4 Electric potential from a plane of charge with surface charge density σ on the xy plane

$$V(x, y, z) = -\frac{\sigma}{2\epsilon_0} |z|$$

1.1.5 $f(x, y, z) = x^2 + y^2 + z^2$

1.1.6 $f(x, y, z) = 4x^2y^3z^4$

1.2 Scalar fields expressed in cylindrical coordinates

1.2.1 $f(s, \phi, z) = s^2 z \cos\phi$

1.2.2 Electric potential produced by a point charge q at the origin

$$V(s, \phi, z) = \frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{s^2 + z^2}}$$

1.2.3 $f(s, \phi, z) = \ln(s/a)$

where a is a constant with dimensions of length.

1.2.4 Magnitude of electric field from a long line charge with linear charge density λ along the z axis

$$E(s, \phi, z) = \frac{\lambda}{2\pi\epsilon_0 s}$$

1.3 Scalar fields expressed in spherical coordinates

1.3.1 $f(r, \theta, \phi) = 1/r^2$

1.3.2 Electric potential produced by a point charge q at the origin

$$V(r, \theta, \phi) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

1.3.3 $f(r, \theta, \phi) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$

where a is a constant with dimensions of length.

1.3.4 $f(r, \theta, \phi) = r^2(3\cos^2\theta - 1)$

1.3.5 $f(r, \theta, \phi) = r^2 \sin\theta \cos\phi$

1.4 Scalar fields expressed in a coordinate-independent fashion

1.4.1 Electric potential produced by a point charge q at the origin

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{|\mathbf{r}|}$$

1.4.2 $f(\mathbf{r}) = \mathbf{b} \cdot \mathbf{r}$

where $\mathbf{b}$ is a constant vector.

1.4.3 $f(\mathbf{r}) = \mathbf{r} \cdot \mathbf{r}$

1.4.4 Electric potential produced by an ideal dipole with dipole moment $\mathbf{p}$ at the origin

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \mathbf{r}}{|\mathbf{r}|^3}$$

2 Vector Fields

2.1 Vector fields expressed in Cartesian coordinates

2.1.1 $\mathbf{F}(x, y, z) = xy\hat{\mathbf{x}} + yz\hat{\mathbf{y}} + yz\hat{\mathbf{z}}$

2.1.2 Electric field from a plane of charge with surface charge density σ on the xy plane

$$\mathbf{E}(x, y, z) = \begin{cases} \frac{\sigma}{2\epsilon_0}\hat{\mathbf{z}} & , \quad z > 0 \\ -\frac{\sigma}{2\epsilon_0}\hat{\mathbf{z}} & , \quad z < 0 \end{cases}$$

2.1.3 Electric field produced by a point charge q at the origin

$$\mathbf{E}(x, y, z) = \frac{q}{4\pi\epsilon_0} \frac{x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}}{(x^2 + y^2 + z^2)^{3/2}}$$

2.1.4 $\mathbf{F}(x, y, z) = x^2\hat{\mathbf{x}} + y^2\hat{\mathbf{y}} + z^2\hat{\mathbf{z}}$

2.1.5 $\mathbf{F}(x, y, z) = xy^2\hat{\mathbf{x}} + xy^2\hat{\mathbf{y}} + 3a^3\hat{\mathbf{z}}$

where a is a constant with dimensions of length.

2.1.6 Electric field produced inside a planar slab by a uniform volume charge density ρ_0

$$\mathbf{E}(x, y, z) = \frac{\rho_0}{\epsilon_0} z\hat{\mathbf{z}}$$

2.1.7 Electric field from a plane of charge with surface charge density σ_0 on the yz plane

$$\mathbf{E}(x, y, z) = \begin{cases} \frac{\sigma_0}{2\epsilon_0}\hat{\mathbf{x}} & , \quad x > 0 \\ -\frac{\sigma_0}{2\epsilon_0}\hat{\mathbf{x}} & , \quad x < 0 \end{cases}$$

2.2 Vector fields expressed in cylindrical coordinates

2.2.1 Electric field from a long line charge with linear charge density λ along the z axis

$$\mathbf{E}(s, \phi, z) = \frac{\lambda}{2\pi\epsilon_0 s}\hat{\mathbf{s}}$$

2.2.2 Electric field from a plane of charge with surface charge density σ on the $z = 0$ plane

$$\mathbf{E}(s, \phi, z) = \begin{cases} \frac{\sigma}{2\epsilon_0}\hat{\mathbf{z}} & , \quad z > 0 \\ -\frac{\sigma}{2\epsilon_0}\hat{\mathbf{z}} & , \quad z < 0 \end{cases}$$

2.2.3 $\mathbf{F}(s,\phi,z) = s^2\hat{\mathbf{s}}$

2.2.4 $\mathbf{F}(s,\phi,z) = s^2\hat{\boldsymbol{\phi}}$

2.2.5 $\mathbf{F}(s,\phi,z) = sz\hat{\mathbf{s}} + s^2\sin\phi\hat{\mathbf{z}}$

2.2.6 $\mathbf{F}(s,\phi,z) = s^2\cos\phi\hat{\mathbf{s}} + s^2\sin\phi\hat{\boldsymbol{\phi}}$

2.2.7 Electric field produced inside a long cylinder by a uniform volume charge density ρ_0

$$\mathbf{E}(s,\phi,z) = \frac{\rho_0}{2\epsilon_0}s\hat{\mathbf{s}}$$

2.3 Vector fields expressed in spherical coordinates

2.3.1 Electric field produced by a point charge q at the origin

$$\mathbf{E}(r,\theta,\phi) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

2.3.2 Electric field produced by an ideal dipole at the origin with dipole moment p in the z direction

$$\mathbf{E}(r,\theta,\phi) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}})$$

2.3.3 $\mathbf{F}(r,\theta,\phi) = r\hat{\mathbf{r}} + r\sin\theta\hat{\boldsymbol{\theta}} + r\sin\theta\cos\phi\hat{\boldsymbol{\phi}}$

2.3.4 $\mathbf{F}(r,\theta,\phi) = r^2\hat{\mathbf{r}} + r^2\sin\theta\cos\theta\hat{\boldsymbol{\theta}} + r^2\sin^2\theta\cos^2\phi\hat{\boldsymbol{\phi}}$

2.3.5 Electric field produced inside a sphere by a uniform volume charge density ρ_0

$$\mathbf{E}(r,\theta,\phi) = \frac{\rho_0}{3\epsilon_0}r\hat{\mathbf{r}}$$

2.3.6 $\mathbf{F}(r, \theta, \phi) = r \sin\theta \hat{\boldsymbol{\theta}}$

2.3.7 $\mathbf{F}(r, \theta, \phi) = r \sin\theta \hat{\boldsymbol{\phi}}$

2.4 Vector fields expressed in a coordinate-independent fashion

2.4.1 Electric field produced by a point charge q at the origin

$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0} \frac{\mathbf{r}}{|\mathbf{r}|^3}$$

2.4.2 Electric field produced by a point charge q_1 at position $\mathbf{r}_1$

$$\mathbf{E}(\mathbf{r}) = \frac{q_1}{4\pi\epsilon_0} \frac{\mathbf{r} - \mathbf{r}_1}{|\mathbf{r} - \mathbf{r}_1|^3}$$

2.4.3 $\mathbf{F}(\mathbf{r}) = 3\mathbf{r}$

2.4.4 $\mathbf{F}(\mathbf{r}) = (\mathbf{r} \cdot \mathbf{r})\mathbf{r}$

3 Paths

3.1 Paths expressed in Cartesian coordinates

3.1.1 The path from $(0, 0, 0)$ to $(x_0, 0, 0)$ along the x axis, then to $(x_0, y_0, 0)$ along a straight line parallel to the y axis, then to (x_0, y_0, z_0) along a straight line parallel to the z axis

3.1.2 The path from $(0, 0, 0)$ to $(0, 0, z_0)$ along the z axis, then to $(0, y_0, z_0)$ along a straight line parallel to the y axis, then to (x_0, y_0, z_0) along a straight line parallel to the x axis

3.1.3 The straight-line path from $(0, 0, 0)$ to (a, a, a)

where a is a constant with dimensions of length.

3.1.4 The path around a square from $(0, 0, 0)$ to $(a, 0, 0)$ to $(a, a, 0)$ to $(0, a, 0)$ to $(0, 0, 0)$

where a is a constant with dimensions of length.

3.1.5 The boundary of surface 4.1.2

3.1.6 The straight-line path from $(0, 0, 3)$ to $(2, 4, 3)$

3.1.7 The path from $(1, 2, 3)$ to $(4, 5, 3)$ that runs from $(1, 2, 3)$ to $(4, 2, 3)$ along a straight line and then from $(4, 2, 3)$ to $(4, 5, 3)$ along a straight line

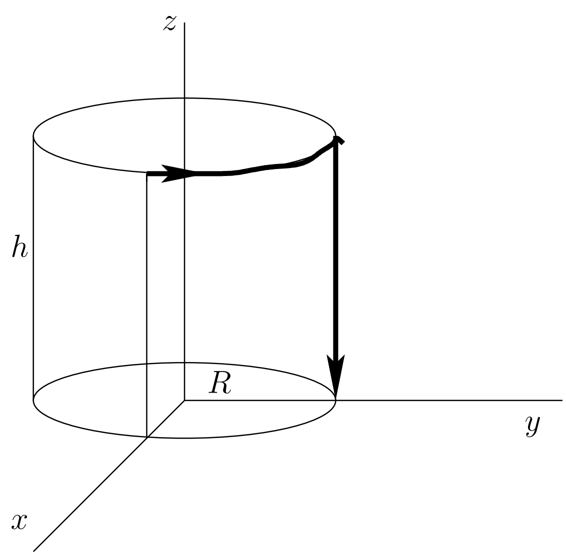
3.2 Paths expressed in cylindrical coordinates

3.2.1 The circular path with radius R in the $z = 0$ plane starting at $\phi = 0$ and going to $\phi = 2\pi$

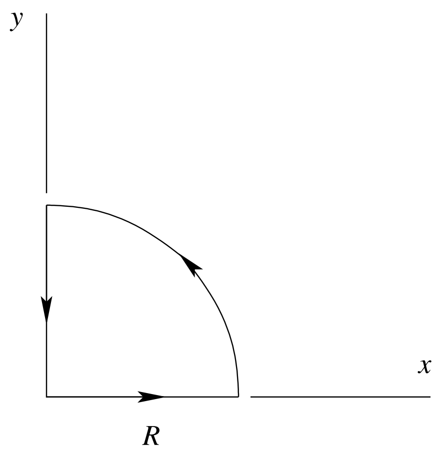
3.2.2 The straight path in the $z = 0$ plane from the origin along $\phi = \pi/4$ until $s = R$

3.2.3 The boundary of surface 4.2.1

3.2.4 the path on the surface of a cylinder of radius R that goes (i) along a circular arc from a point at $(x, y, z) = (R, 0, h)$ to a point at $(x, y, z) = (0, R, h)$, and then (ii) along a straight-line path from $(x, y, z) = (0, R, h)$ to $(x, y, z) = (0, R, 0)$



3.2.5 The closed path shown in the figure



3.3 Paths expressed in spherical coordinates

3.3.1 The path on the surface of a sphere of radius R that goes (i) from the north pole at $(x, y, z) = (0, 0, R)$ to the equator at $(x, y, z) = (R, 0, 0)$, and then (ii) along the equator to the point $(x, y, z) = (R/\sqrt{2}, R/\sqrt{2}, 0)$

3.3.2 The straight-line path from the origin to the point with spherical coordinates $(r, \theta, \phi) = (2, \pi/6, \pi/4)$.

3.3.3 The path from $(r, \theta, \phi) = (R, \pi/4, 0)$ to $(r, \theta, \phi) = (R, \pi/4, \pi/2)$ along which $r = R$ and $\theta = \pi/4$.

3.3.4 The path from $(r, \theta, \phi) = (R, 0, 2\pi/3)$ to $(r, \theta, \phi) = (R, \pi/2, 2\pi/3)$ along which $r = R$ and $\phi = 2\pi/3$.

4 Surfaces

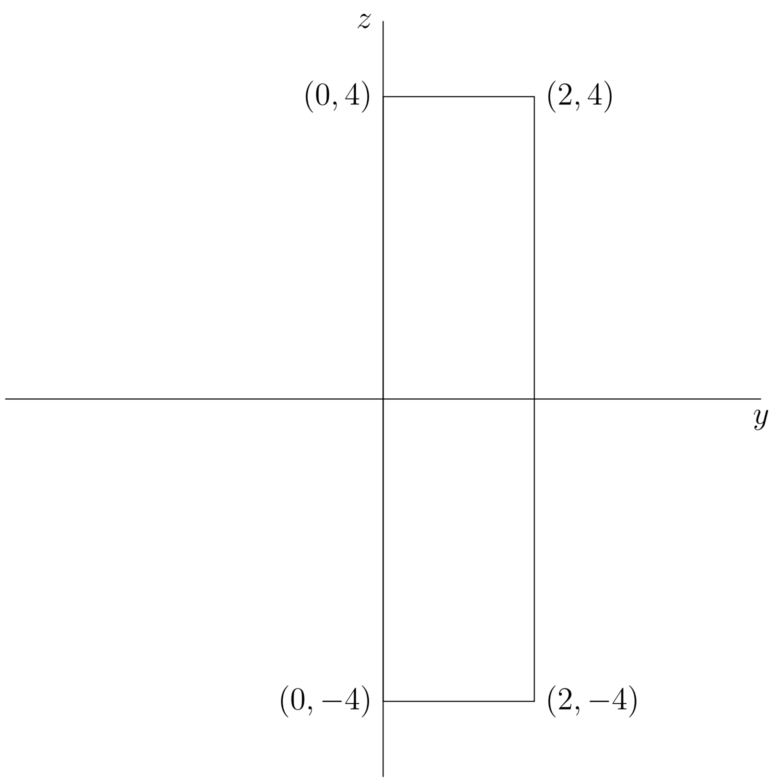
4.1 Surfaces expressed in Cartesian coordinates

4.1.1 The boundary of volume 5.1.1

4.1.2 The region enclosed by a square in the xy plane with side length L centered at the origin

If an orientation is needed, use $\hat{\mathbf{z}}$.

4.1.3 The rectangular region shown in the figure



If an orientation is needed, use $\hat{\mathbf{x}}$.

4.1.4 The region enclosed by a triangle in the xy plane with vertices at $(x, y, z) = (0, 0, 0)$, $(x, y, z) = (2, 0, 0)$, and $(x, y, z) = (2, 2, 0)$

4.1.5 The rectangle with vertices $(x, y, z) = (a, b, c)$, $(x, y, z) = (-a, b, c)$, $(x, y, z) = (-a, -b, c)$, and $(x, y, z) = (a, -b, c)$

The constants a, b, and c are all positive. If an orientation is needed, use $\hat{\mathbf{z}}$.

4.1.6 The rectangle with vertices $(x, y, z) = (a, b, c)$, $(x, y, z) = (a, -b, c)$, $(x, y, z) = (a, -b, -c)$, and $(x, y, z) = (a, b, -c)$

The constants a, b, and c are all positive. If an orientation is needed, use $\hat{\mathbf{x}}$.

4.2 Surfaces expressed in cylindrical coordinates

4.2.1 A disk in the $z = 0$ plane with radius R, centered at the origin

If an orientation is needed, use $\hat{\mathbf{z}}$.

4.2.2 The boundary of volume 5.2.2

4.3 Surfaces expressed in spherical coordinates

4.3.1 A sphere of radius R centered at the origin

5 Volumes

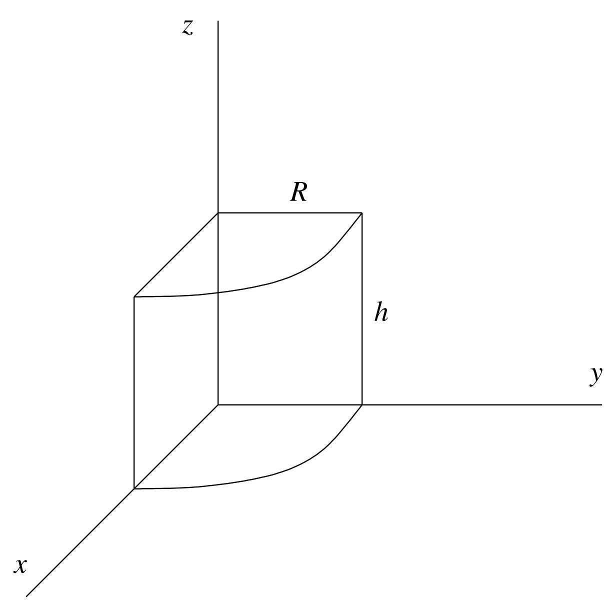
5.1 Volumes expressed in Cartesian coordinates

5.1.1 A cube with side length L centered at the origin

5.1.2 The rectangular box in which $0 \leq x \leq a$, $0 \leq y \leq b$, and $0 \leq z \leq c$, where a, b, and c are some constants with dimensions of length

5.2 Volumes expressed in cylindrical coordinates

5.2.1 The quarter cylinder with height h and radius R shown in the diagram below



5.2.2 A cylinder with height h and radius R centered at the origin, with the cylinder axis on the z axis.

5.3 Volumes expressed in spherical coordinates

5.3.1 A ball of radius R centered at the origin

5.3.2 The upper half of a ball of radius 2, with the center of the ball at the origin