Electricity and Magnetism I (PHY 321)

Coulomb's Law problems

Problem 1 A particle with charge q is stationary at the origin. The electric field produced by this particle is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

Write the electric field in Cartesian components. In other words, give expressions for E_x, E_y, and E_z in terms of x, y, and z.

Problem 2 Consider a 2-nC point charge at $\mathbf{r} = (3\text{ m})\hat{\mathbf{i}}+(4\text{ m})\hat{\mathbf{j}}-(2\text{ m})\hat{\mathbf{k}}$. Compute the x-, y-, and z-components of the electric field produced by this charge at $\mathbf{r} = (1\text{ m})\hat{\mathbf{i}} + (6\text{ m})\hat{\mathbf{j}} - (1\text{ m})\hat{\mathbf{k}}$. Also find the magnitude of the electric field at this point.

Problem 3 Consider a 2.197-μC point charge at $\mathbf{r} = (3\text{ m})\hat{\mathbf{i}} + (4\text{ m})\hat{\mathbf{j}} - (7\text{ m})\hat{\mathbf{k}}$. Compute the x-, y-, and z-components of the electric field produced by this charge at $\mathbf{r} = -(1\text{ m})\hat{\mathbf{i}} + (1\text{ m})\hat{\mathbf{j}} + (5\text{ m})\hat{\mathbf{k}}$. Also find the magnitude of the electric field at this point. Give numerical results with appropriate units.

Problem 4 Write an expression for the electric field $\mathbf{E}(\mathbf{r})$ at a point $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ produced by a point charge q located at $\mathbf{r}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$.

Problem 5 Write an expression for the electric field $\mathbf{E}(\mathbf{r})$ at a point $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ produced by a point charge q_1 located at $\mathbf{r}_1 = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}}$ and a point charge q_2 located at $\mathbf{r}_2 = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}}$.

Problem 6 Consider a 2-nC point charge at $\mathbf{r} = (3\text{ m})\hat{\mathbf{i}}+(4\text{ m})\hat{\mathbf{j}}-(2\text{ m})\hat{\mathbf{k}}$ and a -3-nC point charge at $\mathbf{r} = (8\text{ m})\hat{\mathbf{j}} - (3\text{ m})\hat{\mathbf{k}}$. Compute the x-, y-, and z-components of the electric field produced by these charges at $\mathbf{r} = (1\text{ m})\hat{\mathbf{i}} + (6\text{ m})\hat{\mathbf{j}} - (1\text{ m})\hat{\mathbf{k}}$. Also find the magnitude of the electric field at this point.

Problem 7 Consider a point particle with charge q at $\mathbf{r} = a\hat{\mathbf{k}}$ and another point particle with charge q at $\mathbf{r} = -a\hat{\mathbf{k}}$. Compute the x-, y-, and z-components of the electric field produced by these charges at $\mathbf{r} = x\hat{\mathbf{i}}$. Also find the magnitude of the electric field at this point.

Problem 8 Consider a physical (rather than ideal) electric dipole made up of two point charges. There is a point charge q located at $\mathbf{r}_1 = \frac{a}{2}\hat{\mathbf{k}}$ and a second point charge $-q$ located at $\mathbf{r}_2 = -\frac{a}{2}\hat{\mathbf{k}}$.

(a) Find the electric field $\mathbf{E}(\mathbf{r})$ produced by these two charges at an arbitrary point $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$.

(b) Your expression for the electric field should simplify if we restrict our attention to the electric field in the xy-plane. Give a simplified expression for $\mathbf{E}(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})$.

(c) Using your original expression for the electric field at an arbitrary point in space, find the electric field produced by an ideal dipole as follows. First, using the expression for the electric dipole moment $p = qa$, replace all occurrences of q in your expression for the electric field. Next, take the limit as $a \to 0$.

Problem 9 Write an expression for the electric field $\mathbf{E}(x\hat{\mathbf{i}})$ at a point $\mathbf{r} = x\hat{\mathbf{i}}$ produced by an infinite line charge with uniform linear charge density λ lying on the z-axis.

Problem 10 Write an expression for the electric field $\mathbf{E}(\mathbf{r})$ at a point $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ produced by an infinite line charge with uniform linear charge density λ lying on the z-axis.

Problem 11 Write an expression for the electric field $\mathbf{E}(\mathbf{r})$ at a point $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ produced by an infinite line charge with uniform linear charge density λ lying on the x-axis.

Problem 12 Consider a circular loop of charge, with uniform linear charge density λ_0 and radius R. Suppose the circular loop lies in the xy plane, centered at the origin. Find the electric field $\mathbf{E}(z\hat{\mathbf{k}})$ at any point on the z axis.

Problem 13 Consider a circular disk of charge, with uniform surface charge density σ_0 and radius R. Suppose the circular disk lies in the xy plane, centered at the origin. Find the electric field $\mathbf{E}(z\hat{\mathbf{k}})$ at any point on the z axis.

Problem 14 Consider a 3-C point charge at $\mathbf{r} = (4\text{ m})\hat{\mathbf{y}}$. Find the electric field at $\mathbf{r} = (1\text{ m})\hat{\mathbf{x}} - (1\text{ m})\hat{\mathbf{y}} + (1\text{ m})\hat{\mathbf{z}}$ and at $\mathbf{r} = -(1\text{ m})\hat{\mathbf{x}} - (2\text{ m})\hat{\mathbf{y}} - (3\text{ m})\hat{\mathbf{z}}$. In each case give the components of the electric field as well as the magnitude of the electric field.

Problem 15 Give an expression for the electric field at point $\mathbf{r}$ produced by a point charge q_1 at $\mathbf{r}'_1$ and a point charge q_2 at $\mathbf{r}'_2$. Evaluate your expression if $q_1 = 2$ C, $q_2 = -3$ C, $x = -2$ m, $y = -2$ m, $z = -2$ m, $x'_1 = -3$ m, $y'_1 = -2$ m, $z'_1 = -6$ m, $x'_2 = 1$ m, $y'_2 = -2$ m, and $z'_2 = -2$ m.

Problem 16 Consider a point particle with charge -24 nC at position $(3\text{ m})\hat{\mathbf{i}} + (4\text{ m})\hat{\mathbf{j}} + (5\text{ m})\hat{\mathbf{k}}$ and a second point particle with charge 12 nC at position $(5\text{ m})\hat{\mathbf{i}} - (4\text{ m})\hat{\mathbf{j}} + (7\text{ m})\hat{\mathbf{k}}$. Find the electric field $\mathbf{E}$ produced by these two point particles at the position $(1\text{ m})\hat{\mathbf{i}} + (9\text{ m})\hat{\mathbf{k}}$. Give a numerical answer with appropriate units.

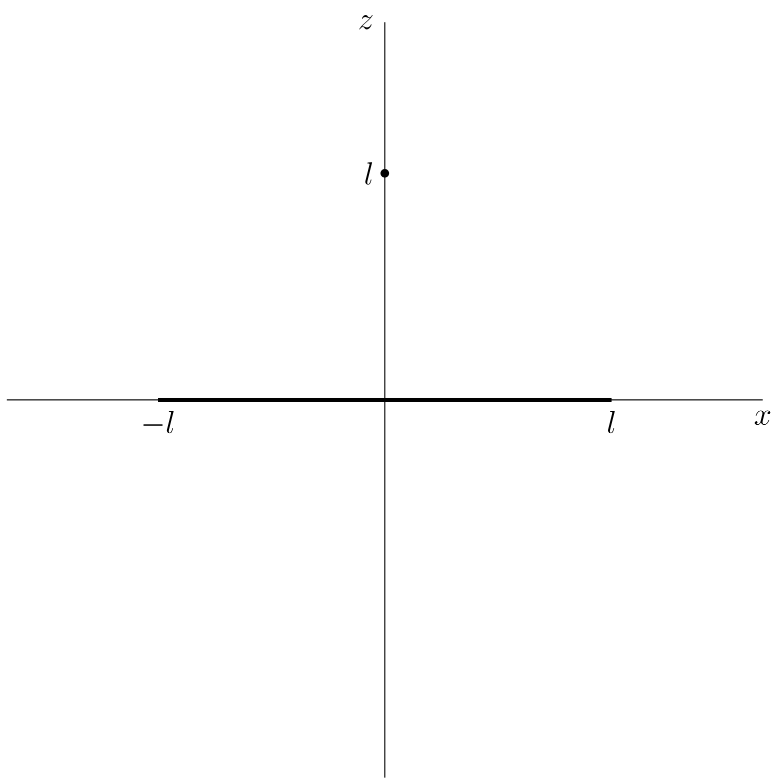
Problem 17 Consider a charged spherical ball with radius R. A total charge Q is distributed uniformly throughout the volume of the ball. Using a coordinate system in which the origin is at the center of the ball, find the electric field produced by this ball at position $y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$. Give separate expressions for E_x, E_y, and E_z. Your expressions should include integrals with appropriate limits, without any unit vectors. You do not need to evaluate any of these integrals, but if you can argue that an expression should be zero on physical or mathematical grounds, please do so.

Problem 18 Consider a charged cylindrical pellet with radius R and height h. A total charge Q is distributed uniformly throughout the volume of the cylinder. Using a coordinate system in which the z axis coincides with the axis of the cylindrical pellet, and the origin is at the center of the cylinder, find the electric field produced by this pellet at position $x\hat{\mathbf{i}} + z\hat{\mathbf{k}}$. Give separate expressions for E_x, E_y, and E_z. Your expressions should include integrals with appropriate limits, without any unit vectors. You do not need to evaluate any of these integrals, but if you can argue that an expression should be zero on physical or mathematical grounds, please do so.

Problem 19 Consider a uniformly charged ball with radius R and total charge $2Q$, centered at the origin of our coordinate system. Imagine that we slice the ball in half along the $z = 0$ plane, and throw the portion below the xy plane away (below the xy plane meaning in the region $z < 0$). What is left is half of a ball with radius R and total charge Q, centered at the origin and living in the region $z > 0$. Find the electric field produced by this half ball at position $x\hat{\mathbf{i}} + z\hat{\mathbf{k}}$. Give separate expressions for E_x, E_y, and E_z. Your expressions should include integrals with appropriate limits, without any unit vectors. You do not need to evaluate any of these integrals, but if you can argue that an expression should be zero on physical or mathematical grounds, please do so.

Problem 20 Consider a straight segment of wire with length L. The segment is located along the z-axis, with one end at $\mathbf{r} = \frac{L}{2}\hat{\mathbf{k}}$ and the other end at $\mathbf{r} = -\frac{L}{2}\hat{\mathbf{k}}$. There is a uniform line charge density λ_0 on this wire segment. Find the electric field at an arbitrary point $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$.

Problem 21 (from Fall 2014 Exam 1) Consider a charged filament (line segment) lying along the x axis from $x = -l$ to $x = l$, with uniform linear charge density λ. Find the electric field $\mathbf{E}$ produced by the filament at a point on the positive z axis a distance l from the center of the filament.



Problem 22 (from Fall 2017 Exam 1) A thin rod with length L lies on the x axis from $x = 0$ to $x = L$. The rod has a total charge Q, uniformly distributed along its length. Find the electric field (magnitude and direction) on the x axis at $x = 2L$.

Problem 23 A thin rod with length L lies on the x axis from $x = 0$ to $x = L$. The rod has a total charge Q, uniformly distributed along its length. Find the electric field (x and y components) at the point where $x = L$ and $y = L$. Also find the magnitude of the electric field at this point. Compare this magnitude to the electric field of a point charge at $(0, 0)$ and to the electric field of a point charge at $(0, L)$. It should be greater than the former and smaller than the latter. (Do you see why?)

Problem 24 A thin rod with length L lies on the x axis from $x = 0$ to $x = L$. The rod has a total charge Q, uniformly distributed along its length. Find the electric field at the point where $x = L/2$ and $y = L$.

Problem 25 A thin circular hoop with radius R has charge Q uniformily distributed over its length. The electric field at the center of the hoop is zero because the contribution from each charged element of the hoop is canceled by the contribution from an element directly across the hoop. (a) Suppose that we carefully cut the hoop in half, and remove one of the halves, so that we are left with a semicircular hoop with charge $Q/2$. Now what is the electric field at the center? (b) Suppose that we carefully cut the half hoop in half, and remove one of the pieces, so that we are left with a quarter-circular hoop with charge $Q/4$. Now what is the electric field at the center?

Problem 26 A model for an electric dipole is a stick of length L lying on the z axis from $z = -L/2$ to $z = L/2$. The stick has a linear charge density that changes with position.

$$\lambda(z) = 2\lambda_0 \frac{z}{L}$$

(a) Find the electric field at $(x, y, z) = (L, 0, 0)$. (b) Find the electric field at $(x, y, z) = (0, 0, L)$.