Electricity and Magnetism I (PHY 321)

Gauss's Law problems

Problem 1 (Fleisch Problem 1.1) Find the electric flux through the surface of a sphere containing 15 protons and 10 electrons. Does the size of the sphere matter?

Problem 2 (Fleisch Problem 1.3) Find the total electric flux through a closed cylinder containing a line charge along its axis with linear charge density $\lambda = \lambda_0(1 - x/h)$ if the cylinder and the line charge extend from $x = 0$ to $x = h$.

Problem 3 (Fleisch Problem 1.4) What is the flux through any closed surface surrounding a charged sphere of radius a_0 with volume charge density of $\rho = \rho_0(r/a_0)$, where r is the distance from the center of the sphere?

Problem 4 (Fleisch Problem 1.5) A circular disk with surface charge density 2×10^{-10} C/m^2 is surrounded by a sphere with radius of 1 m. If the flux through the sphere is 5.2×10^{-2} V m, what is the diameter of the disk?

Problem 5 Find the electric field produced by a point charge q at the origin (using Gauss's law).

Problem 6 Consider a uniformly charged spherical ball with radius R and total charge Q. The charge is uniformly distributed throughout the volume of the ball. Find the electric field everywhere.

Problem 7 Consider a uniformly charged sphere with radius R and total charge Q. The charge is uniformly distributed across the surface of the sphere. Find the electric field everywhere.

Problem 8 Consider a fat spherical shell with inner radius a and outer radius b. There is charge distributed in the region $a < r < b$. The total charge is Q, and the charge density is proportional to the square of the distance from the origin. Find the electric field everywhere.

Problem 9 A hydrogen atom in its ground state is made up of a stationary proton at the origin and an electron cloud with a charge density

$$\rho(r, \theta, \phi) = -\frac{e}{\pi a_0^3} e^{-2r/a_0},$$

where e is the charge of the proton and a_0 is the Bohr radius. Find the electric field produced by the hydrogen atom at an arbitrary point in space. Express your answer in terms of e and a_0.

Problem 10 Consider a long straight line charge with linear charge density λ. Choose a coordinate system and find the electric field produced by this line charge at an arbitrary point in space.

Problem 11 Consider an infinitely long cylindrical shell with radius R and uniform surface charge density σ. Find the electric field inside and outside of the shell.

Problem 12 Consider a cylinder with radius R, infinitely long. A uniform charge density ρ is distributed throughout the volume of the cylinder. Take the z axis of your coordinate system to be along the axis of the cylinder, and find the electric field at an arbitrary point in space, inside or outside of the cylinder.

Problem 13 Consider an infinitely long cylinder with radius a. The charge density of the cylinder increases linearly with distance from the central axis, $\rho(s) = ks$, for some constant k with appropriate units. (Here, s is the distance from the central axis.) The charge density outside the cylinder is zero. Find the electric field inside and outside of the cylinder.

Problem 14 Consider two infinitely long concentric cylindrical shells. The inner shell has radius a and the outer shell has radius b. The inner shell has a uniform charge distributed over its surface with a total charge per unit length of λ. The outer shell has a uniform charge distributed over its surface with a total charge per unit length of $-\lambda$. Find the electric field everywhere (there are three regions to be concerned with).

Problem 15 Consider an infinite plate of charge with surface charge density σ. Find the electric field everywhere.

Problem 16 A parallel-plate capacitor consists of two infinite planes of charge separated by a distance d. One plane has surface charge density σ and the other has surface charge density $-\sigma$. Find the electric field everywhere. (Hint: Use the result for the electric field produced by a single infinite plate of charge.)

Problem 17 Consider a thick, infinite plate of charge. The plate has thickness d and uniform charge density ρ. Choose a coordinate system and find the electric field everywhere.

Problem 18 Consider a uniformly charged plate at $z = 0$ with surface charge density σ_1 and a second uniformly charged plate at $z = d$ with surface charge density σ_2. Find the electric field at $x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ produced by these plates.

Problem 19 Consider an electric field

$$\mathbf{E} = as^2\hat{\mathbf{s}} + bz^2\hat{\mathbf{z}},$$

where a and b are constants with appropriate units.

(a) Find the volume charge density that would produce this electric field.

(b) Consider a cylinder with radius R and height $2L$ centered at the origin (the top of the cylinder lies in the plane $z = L$ and the bottom of the cylinder lies in the plane $z = -L$). Find the electric flux through the top disk, bottom disk, and cylindrical center of the cylinder.

(c) Find the electric flux through the entire closed surface of the cylinder.

(d) Find the charge enclosed by the cylinder in two different ways.

Problem 20 Consider a fat spherical shell with inner radius a and outer radius b. There is charge distributed in the region $a < r < b$. The total charge is Q, and the volume charge density is uniform in the region $a < r < b$. Find the electric field everywhere.

Problem 21 (from Fall 2014 Exam 1) Consider the following point charges, with locations given in Cartesian coordinates.

Charge	(x, y, z)
2.00 nC	(0.00 m, 0.00 m, 0.00 m)
−5.00 nC	(3.00 m, 3.00 m, 3.00 m)
8.00 nC	(3.00 m, 0.00 m, −3.00 m)
−16.00 nC	(−1.00 m, −2.00 m, −3.00 m)

Find the electric flux produced by these four point charges through a sphere of radius 4 m centered at the origin.