Electricity and Magnetism I (PHY 321)

Lorentz Force Law problems

Problem 1 A particle with mass m and charge q is in an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$. Use Newton's second law to write a vector differential equation for the velocity $\mathbf{v}(t)$ of the particle. Write it as

$$\frac{d\mathbf{v}}{dt} =$$

Then take Cartesian components and write expressions for

$$\frac{dv_x}{dt} = \qquad (1)$$

$$\frac{dv_y}{dt} = \qquad (2)$$

$$\frac{dv_z}{dt} = \qquad (3)$$

in terms of the eleven quantities $m, q, v_x, v_y, v_z, E_x, E_y, E_z, B_x, B_y, B_z$. Finally, say for each of the eleven quantities whether it is a constant, a function of time (t), or a function of space-time (x, y, z, and t).

Problem 2 Consider a classical hydrogen atom consisting of a proton fixed at the origin and an electron free to move. Let $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ denote the position of the electron.

1. Write an expression for the force $\mathbf{F}$ that the electron feels by being near the proton.
2. Write down Cartesian components of equations of motion for the electron in the form

$$\frac{d^2x}{dt^2} = \qquad (4)$$

$$\frac{d^2y}{dt^2} = \qquad (5)$$

$$\frac{d^2z}{dt^2} = \quad . \qquad (6)$$

in terms of x, y, z and whatever else you need (but force, electric field, and electric potential should not appear explicitly).

Problem 3 A particle with charge q and mass m passes through the origin of a system of coordinates at time $t = 0$ with velocity $\mathbf{v} = v_0\hat{\mathbf{k}}$. A uniform electric field $\mathbf{E}(\mathbf{r}) = E_0\hat{\mathbf{k}}$ and a uniform magnetic field $\mathbf{B}(\mathbf{r}) = B_0\hat{\mathbf{k}}$ are present in the vicinity of the of the charged particle. (This electric field and magnetic field are produced by *other* charges and currents.) Give expressions for the position $\mathbf{r}(t)$ and the velocity $\mathbf{v}(t)$ of the particle as functions of time.

Problem 4 Consider an electric field

$$\mathbf{E}(\mathbf{r}) = \begin{cases} E_0\hat{\mathbf{k}} & , \quad x < a \\ 0 & , \quad x \geq a \end{cases}$$

where $a > 0$. A particle with charge q, mass m, initial position $\mathbf{r}(0) = 0$, and initial velocity $\mathbf{v}(0) = v_0\hat{\mathbf{i}}$ travels through the electric field. (v_0 is a constant with units of velocity.) Find the position $\mathbf{r}(t)$ and velocity $\mathbf{v}(t)$ of the particle as a function of time. There is no magnetic field present.

Problem 5 Consider a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{k}}$, and no electric field. A particle with charge q and mass m begins in this magnetic field with a position $\mathbf{r}(0) = 0$ and a velocity $\mathbf{v}(0) = v_0\hat{\mathbf{i}}$. Find the position $\mathbf{r}(t)$ and the velocity $\mathbf{v}(t)$ of the particle at a later time t.

Problem 6 Consider a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{x}}$ and a uniform electric field $\mathbf{E} = E_0\hat{\mathbf{z}}$. A particle with charge q and mass m begins in this electromagnetic field with a position $\mathbf{r}(0) = 0$ and a velocity $\mathbf{v}(0) = v_0\hat{\mathbf{y}}$.

(a) Find the velocity $\mathbf{v}(t)$ of the particle at a later time t.

(b) Find $v_x(t)$, $v_y(t)$, and $v_z(t)$ for the particle in Example 5.2 of Griffiths (4th ed). If $v_0 = 0$, the result of part (a) should agree with that of Example 5.2. Confirm that this is true.

(c) There is a special value of v_0 which will cause the particle to travel in a straight line at constant velocity. Find this value of v_0.

Problem 7 A particle with mass m and charge q moves in a region of space in which there is a uniform electric field $\mathbf{E} = E_0\hat{\mathbf{z}}$ and a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. At time $t = 0$, the particle is passing through the origin with velocity $\mathbf{v}(0) = v_0\hat{\mathbf{z}}$.

(a) Find the velocity $\mathbf{v}(t)$ of the particle as a function of time.

(b) Find the position $\mathbf{r}(t)$ of the particle as a function of time.

Problem 8 A particle with mass m and charge q moves in a region of space in which there is a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. No electric field is present. At time $t = 0$, the particle is passing through the origin with velocity $\mathbf{v}(0) = v_0\hat{\mathbf{x}}$.

(a) Find the velocity $\mathbf{v}(t)$ of the particle as a function of time.

(b) Find the position $\mathbf{r}(t)$ of the particle as a function of time.