

Quantum Mechanics I (PHY 421)

Eigenvalues and Eigenvectors

Problem 1 Find the eigenvalues of the following matrix.

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

For each eigenvalue, find a normalized eigenvector.

Problem 2 Find the eigenvalues of the following matrix.

$$\begin{bmatrix} -2 & 4 \\ 4 & 4 \end{bmatrix}$$

For each eigenvalue, find a normalized eigenvector.

Problem 3 Find the eigenvalues of the following matrix. The eigenvalues are integers, so you might be able to guess the roots of the cubic characteristic equation.

$$\begin{bmatrix} -2 & 4 & 0 \\ 4 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Each eigenvalue gives rise to an eigenspace. (The eigenspace is the collection of eigenvectors with that eigenvalue). Give an orthonormal basis for each eigenspace.

Problem 4 Find the eigenvalues of the following matrix. The eigenvalues are integers, so you might be able to guess the roots of the cubic characteristic equation.

$$\begin{bmatrix} 5 & 0 & 2 \\ 0 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

Each eigenvalue gives rise to an eigenspace. (The eigenspace is the collection of eigenvectors with that eigenvalue). Give an orthonormal basis for each eigenspace.

Problem 5 Find the eigenvalues of the following matrix. The eigenvalues are integers, so you might be able to guess the roots of the cubic characteristic equation.

$$\begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

Each eigenvalue gives rise to an eigenspace. (The eigenspace is the collection of eigenvectors with that eigenvalue). Give an orthonormal basis for each eigenspace.