

Quantum Mechanics I (PHY 421)

Fall 2011

Exam 2

Problem 1 Consider a spin-1/2 particle in the state

$$|\psi\rangle = \frac{3}{5}|z_+\rangle + \frac{4}{5}|z_-\rangle.$$

- (a) Find $\langle S_x \rangle$.
- (b) Find $\langle S_y \rangle$.
- (c) Find $\langle S_z \rangle$.
- (d) Find ΔS_x .
- (e) Find ΔS_y .
- (f) The uncertainty principle asserts that $\Delta S_x \Delta S_y \geq C$, for some particular value C . Find C . Check that the product of your results in parts (d) and (e) is greater than or equal to C .

Problem 2 An atom has basis states $|0\rangle$, $|1\rangle$, and $|2\rangle$, and a Hamiltonian operator such that

$$\begin{aligned}H|0\rangle &= 0 \\H|1\rangle &= E_0|1\rangle \\H|2\rangle &= 2E_0|2\rangle\end{aligned}$$

where E_0 is a constant with units of energy. Let the initial state of the atom be

$$|\psi(0)\rangle = \frac{1}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle + \frac{1}{\sqrt{3}}|2\rangle.$$

- (a) Calculate $\langle E \rangle$ and ΔE for this state.
- (b) Find an explicit expression for $|\psi(t)\rangle$.
- (c) Let Δt be the smallest length of time for which $\langle \psi(\Delta t) | \psi(0) \rangle = 0$. That is, Δt is the time it takes for the initial state to evolve into something distinguishable from the initial state. Find Δt .
- (d) Calculate $\Delta E \Delta t$.