Quantum Mechanics I (PHY 421)

Spin-1/2 Dynamics Homework

Problem 1 Consider a spin-1/2 particle in a magnetic field. If the magnetic field is in the y-direction, and the initial state of the particle is $|\psi(0)\rangle = |z_+\rangle$, find the state of the particle $|\psi(t)\rangle$ at a later time t. You may take the Hamiltonian to be

$$H = \omega_0 S_y$$

Express your final answer in the $\{|z_+\rangle, |z_-\rangle\}$ basis and simplify as much as possible.

Problem 2 A spin-1/2 particle sits in a magnetic field in the x-direction. At time $t = 0$, the state of the spin-1/2 particle is

$$|\psi(0)\rangle = \frac{\sqrt{3}}{2}|z_+\rangle + \frac{1}{2}|z_-\rangle .$$

1. Find the state of the particle at a later time t.
2. Find $\langle S_x \rangle$ as a function of time.
3. Find $\langle S_y \rangle$ as a function of time.
4. Find $\langle S_z \rangle$ as a function of time.

Problem 3 A spin-1/2 particle emerges from a Stern-Gerlach measurement in the z direction with spin down, and immediately enters a region where there is a uniform magnetic field in the y direction. Taking the Hamiltonian to be

$$H = \omega_0 S_y,$$

Find $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$ as functions of time.