Quantum Mechanics I (PHY 421)

Operators Homework

For a spin-1/2 particle, you know that

$$\begin{aligned}
\mathsf{S}_x |x_+\rangle &= \frac{\hbar}{2} |x_+\rangle \\
\mathsf{S}_x |x_-\rangle &= -\frac{\hbar}{2} |x_-\rangle \\
\mathsf{S}_y |y_+\rangle &= \frac{\hbar}{2} |y_+\rangle \\
\mathsf{S}_y |y_-\rangle &= -\frac{\hbar}{2} |y_-\rangle \\
\mathsf{S}_z |z_+\rangle &= \frac{\hbar}{2} |z_+\rangle \\
\mathsf{S}_z |z_-\rangle &= -\frac{\hbar}{2} |z_-\rangle \\
\mathsf{S}_n |n_+\rangle &= \frac{\hbar}{2} |n_+\rangle \\
\mathsf{S}_n |n_-\rangle &= -\frac{\hbar}{2} |n_-\rangle ,
\end{aligned}$$

where

$$\begin{aligned}
|n_+\rangle &= \cos\frac{\theta}{2} |z_+\rangle + e^{i\phi} \sin\frac{\theta}{2} |z_-\rangle \\
|n_-\rangle &= \sin\frac{\theta}{2} |z_+\rangle - e^{i\phi} \cos\frac{\theta}{2} |z_-\rangle .
\end{aligned}$$

Problem 1 Evaluate the following.

1. $\mathsf{S}_z |y_+\rangle$
2. $\mathsf{S}_z |n_+\rangle$
3. $\mathsf{S}_n |x_+\rangle$

Problem 2 Evaluate the following expressions. These should come out to be complex numbers (not kets, bras, or operators).

1. $\langle x_+| \mathsf{S}_z |y_+\rangle$

2. $\langle y_+ | \, \mathsf{S}_z \, | y_+ \rangle$
3. $\langle z_+ | \, \mathsf{S}_z \, | y_+ \rangle$
4. $\langle y_- | \, \mathsf{S}_z \, | y_+ \rangle$
5. $\langle x_+ | \, \mathsf{S}_n \, | x_+ \rangle$

Problem 3 Consider a spin-1/2 particle in the state

$$|\psi\rangle = \cos\frac{\theta}{2}\,|z_+\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|z_-\rangle\,.$$

Find ΔS_z as a function of θ and ϕ.